Laplace Transform Table

Largely modeled on a table in D'Azzo and Houpis, *Linear Control Systems Analysis and Design*, 1988

$F(s)$	$f(t) \quad 0 \le t$
1. 1	$\delta(t)$ unit impulse at $t = 0$
2. $\frac{1}{s}$	1 or $u(t)$ unit step starting at $t = 0$
3. $\frac{1}{s^2}$	$t \cdot u(t)$ or t ramp function
4. $\frac{1}{s^n}$	$\frac{1}{(n-1)!} t^{n-1}$ n = positive integer
5. $\frac{1}{s} e^{-as}$	$u(t-a)$ unit step starting at $t = a$
6. $\frac{1}{s}(1-e^{-as})$	$u(t)-u(t-a)$ rectangular pulse
7. $\frac{1}{s+a}$	e^{-at} exponential decay
8. $\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!} t^{n-1} e^{-at}$ n = positive integer
9. $\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$
10. $\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}(1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt})$
11. $\frac{s+\alpha}{s(s+a)(s+b)}$	$\frac{1}{ab}[\alpha-\frac{b(\alpha-a)}{b-a}e^{-at}+\frac{a(\alpha-b)}{b-a}e^{-bt}]$
12. $\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}(e^{-at}-e^{-bt})$
13. $\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b}(ae^{-at}-be^{-bt})$

$F(s)$	$f(t) \qquad 0 \le t$
14. $\dfrac{s+\alpha}{(s+a)(s+b)}$	$\dfrac{1}{b-a}[(\alpha-a)e^{-at}-(\alpha-b)e^{-bt}]$
15. $\dfrac{1}{(s+a)(s+b)(s+c)}$	$\dfrac{e^{-at}}{(b-a)(c-a)}+\dfrac{e^{-bt}}{(c-b)(a-b)}+\dfrac{e^{-ct}}{(a-c)(b-c)}$
16. $\dfrac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\dfrac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\dfrac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\dfrac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
17. $\dfrac{\omega}{s^2+\omega^2}$	$\sin \ \omega t$
18. $\dfrac{s}{s^2+\omega^2}$	$\cos \ \omega t$
19. $\dfrac{s+\alpha}{s^2+\omega^2}$	$\dfrac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t+\phi) \qquad \phi=\text{atan2}(\omega,\alpha)$
20. $\dfrac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$	$\sin(\omega t+\theta)$
21. $\dfrac{1}{s(s^2+\omega^2)}$	$\dfrac{1}{\omega^2}(1-\cos\omega t)$
22. $\dfrac{s+\alpha}{s(s^2+\omega^2)}$	$\dfrac{\alpha}{\omega^2}-\dfrac{\sqrt{\alpha^2+\omega^2}}{\omega^2}\cos(\omega t+\phi) \qquad \phi=\text{atan2}(\omega,\alpha)$
23. $\dfrac{1}{(s+a)(s^2+\omega^2)}$	$\dfrac{e^{-at}}{a^2+\omega^2}+\dfrac{1}{\omega\sqrt{a^2+\omega^2}}\sin(\omega t-\phi)$ $\phi=\text{atan2}(\omega,\alpha)$
24. $\dfrac{1}{(s+a)^2+b^2}$	$\dfrac{1}{b}e^{-at}\sin(bt)$
24a. $\dfrac{1}{s^2+2\zeta\omega_n s+\omega_n{}^2}$	$\dfrac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t)$
25. $\dfrac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$

$F(s)$	$f(t) \quad 0 \le t$
26. $\dfrac{s+\alpha}{(s+a)^2+b^2}$	$\dfrac{\sqrt{(\alpha-a)^2+b^2}}{b} e^{-at}\sin(bt+\phi) \qquad \phi = \mathrm{atan2}(b, \alpha-a)$
26a. $\dfrac{s+\alpha}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\sqrt{\dfrac{\left(\frac{\alpha}{\omega_n}-\zeta\omega_n\right)^2}{1-\zeta^2}+1} \cdot e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \mathrm{atan2}(\omega_n\sqrt{1-\zeta^2}, \alpha-\zeta\omega_n)$
27. $\dfrac{1}{s[(s+a)^2+b^2]}$	$\dfrac{1}{a^2+b^2}+\dfrac{1}{b\sqrt{a^2+b^2}} e^{-at}\sin(bt-\phi) \qquad \phi = \mathrm{atan2}(b,-a)$
27a. $\dfrac{1}{s(s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{1}{\omega_n^2}-\dfrac{1}{\omega_n^2\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \cos^{-1}\zeta$
28. $\dfrac{s+\alpha}{s[(s+a)^2+b^2]}$	$\dfrac{\alpha}{a^2+b^2}+\dfrac{1}{b}\sqrt{\dfrac{(\alpha-a)^2+b^2}{a^2+b^2}} e^{-at}\sin(bt+\phi)$ $\phi = \mathrm{atan2}(b, \alpha-a) - \mathrm{atan2}(b,-a)$
28a. $\dfrac{s+\alpha}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\dfrac{\alpha}{\omega_n^2}+\dfrac{1}{\omega_n\sqrt{1-\zeta^2}}\sqrt{(\dfrac{\alpha}{\omega_n}-\zeta)^2+(1-\zeta^2)} \cdot e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \mathrm{atan2}(\omega_n\sqrt{1-\zeta^2}, \alpha-\omega_n\zeta) - \mathrm{atan2}(\sqrt{1-\zeta^2}, -\zeta)$
29. $\dfrac{1}{(s+c)[(s+a)^2+b^2]}$	$\dfrac{e^{-ct}}{(c-a)^2+b^2}+\dfrac{e^{-at}\sin(bt-\phi)}{b\sqrt{(c-a)^2+b^2}} \qquad \phi = \mathrm{atan2}(b, c-a)$

F(s)	*f(t)* $0 \le 1$
30. $\dfrac{1}{s(s+c)[(s+a)^2+b^2]}$	$\dfrac{1}{c(a^2+b^2)} - \dfrac{e^{-ct}}{c[(c-a)^2+b^2]} + \dfrac{e^{-at}\sin(bt-\phi)}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}}$ $\phi = \text{atan2}(b,-a) + \text{atan2}(b, c-a)$
31. $\dfrac{s+\alpha}{s(s+c)[(s+a)^2+b^2]}$	$\dfrac{\alpha}{c(a^2+b^2)} + \dfrac{(c-\alpha)e^{-ct}}{c[(c-a)^2+b^2]}$ $+\dfrac{\sqrt{(\alpha-a)^2+b^2}}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}}e^{-at}\sin(bt+\phi)$ $\phi = \text{atan2}(b, \alpha-a) - \text{atan2}(b,-a) - \text{atan2}(b, c-a)$
32. $\dfrac{1}{s^2(s+a)}$	$\dfrac{1}{a^2}(at-1+e^{-at})$
33. $\dfrac{1}{s(s+a)^2}$	$\dfrac{1}{a^2}(1-e^{-at}-ate^{-at})$
34. $\dfrac{s+\alpha}{s(s+a)^2}$	$\dfrac{1}{a^2}[\alpha-\alpha e^{-at}+a(a-\alpha)te^{-at}]$
35. $\dfrac{s^2+\alpha_1 s+\alpha_0}{s(s+a)(s+b)}$	$\dfrac{\alpha_0}{ab} + \dfrac{a^2-\alpha_1 a+\alpha_0}{a(a-b)}e^{-at} - \dfrac{b^2-\alpha_1 b+\alpha_0}{b(a-b)}e^{-bt}$
36. $\dfrac{s^2+\alpha_1 s+\alpha_0}{s[(s+a)^2+b^2]}$	$\dfrac{\alpha_0}{c^2} + \dfrac{1}{bc}[(a^2-b^2-\alpha_1 a+\alpha_0)^2$ $+b^2(\alpha_1-2a)^2]^{\frac{1}{2}}e^{-at}\sin(bt+\phi)$ $\phi = \text{atan2}[b(\alpha_1-2a), a^2-b^2-\alpha_1 a+\alpha_0] - \text{atan2}(b,-a)$ $c^2 = a^2+b^2$

F(s)	*f(t)* $0 \le 1$
37. $\dfrac{1}{(s^2+\omega^2)[(s+a)^2+b^2]}$	$\dfrac{(1/\omega)\sin(\omega t+\phi_1)+(1/b)e^{-at}\sin(bt+\phi_2)}{[4a^2\omega^2+(a^2+b^2-\omega^2)^2]^{\frac{1}{2}}}$ $\phi_1 = \mathrm{atan2}(-2a\omega, a^2+b^2-\omega^2)$ $\phi_2 = \mathrm{atan2}(2ab, a^2-b^2+\omega^2)$
38. $\dfrac{s+\alpha}{(s^2+\omega^2)[(s+a)^2+b^2]}$	$\dfrac{1}{\omega}\left(\dfrac{\alpha^2+\omega^2}{c}\right)^{\frac{1}{2}}\sin(\omega t+\phi_1)$ $+\dfrac{1}{b}\left[\dfrac{(\alpha-a)^2+b^2}{c}\right]^{\frac{1}{2}}e^{-at}\sin(bt+\phi_2)$ $c=(2a\omega)^2+(a^2+b^2-\omega^2)^2$ $\phi_1 = \mathrm{atan2}(\omega,\alpha)-\mathrm{atan2}(2a\omega, a^2+b^2+\omega^2)$ $\phi_2 = \mathrm{atan2}(b,\alpha-a)+\mathrm{atan2}(2ab, a^2-b^2-\omega^2)$
39. $\dfrac{s+\alpha}{s^2[(s+a)^2+b^2]}$	$\dfrac{1}{c}\left(\alpha t+1-\dfrac{2\alpha a}{c}\right)+\dfrac{[b^2+(\alpha-a)^2]^{\frac{1}{2}}}{bc}e^{-at}\sin(bt+\phi)$ $c=a^2+b^2$ $\phi = 2\mathrm{atan2}(b,a)+\mathrm{atan2}(b,\alpha-a)$
40. $\dfrac{s^2+\alpha_1 s+\alpha_0}{s^2(s+a)(s+b)}$	$\dfrac{\alpha_1+\alpha_0 t}{ab}-\dfrac{\alpha_0(a+b)}{(ab)^2}-\dfrac{1}{a-b}\left(1-\dfrac{\alpha_1}{a}+\dfrac{\alpha_0}{a^2}\right)e^{-at}$ $-\dfrac{1}{b-a}\left(1-\dfrac{\alpha_1}{b}+\dfrac{\alpha_0}{b^2}\right)e^{-bt}$